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FACULTY WORKING PAPER NO. 1065

Risk Premia and the Variation of Stock Index Futures

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Risk Premia and the Variation of Stock Index Futures

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ABSTRACT

The pricing of stock index futures is examined by combining a multiperiod asset pricing model with the arbitrage relationship in Cox, Ingersol, and Ross (1981). By positing stochastic processes for stock prices and the marginal utility of wealth, we derive several empirically testable results for stock index futures. We use weekly price changes and we find evidence that there are time-varying risk premia and that the variance changes as the futures contracts approach maturity.

RISK PREMIA AND THE VARIATION OF STOCK INDEX FUTURES

I. Introduction

Since its inception in 1982, trading in stock index futures has attracted the interests of portfolio managers, investors, speculators, and academic researchers. As is the case with other futures markets, we would like to know how prices are determined and how they vary over time.[1] Do the prices satisfy the martingale model or do they incorporate a risk premium? More recently, interest has turned to the variation of futures prices and how it might change as we approach maturity.[2] In some respects, stock index futures are easier to analyze than other futures markets. The contracts expire on specific dates and must be settled in cash; hence the costs for settling a maturing contract are relatively small and there is little confusion during the delivery month. Delivery for commodity futures and Treasury bonds futures is considerably more complicated and numerous arbitrage strategies during the delivery months have been developed. Moreover, stock index futures are important because they have the potential to provide additional information about the stock market and the determination of stock prices.

Much of the recent literature in finance has been directed at studying the behavior of asset prices and futures and forward prices within models which incorporate risk aversion, stochastic interest rates, and stochastic investment opportunity sets.[3] In the case of futures prices, we no longer get a martingale result, even though the martingale model seems to be a good empirical approximation for some markets. In this paper, we extend a recent model for futures prices,

which incorporates risk aversion and stochastic interest rates, and we get a result which is close to the martingale model, but contains some subtle differences. We then apply the model to stock index futures and empirically examine the deviations from the martingale model. We find evidence of time-varying risk premia and variances in our samples of weekly changes in the logarithm of prices on stock index futures. We develop the theoretical model in Section II and present the empirical results in Section III.

II. A Model of Futures Prices

In this section, the behavior of prices on stock index futures is examined within the context of a model with risk aversion and stochastic interest rates. A discrete-time intertemporal asset pricing model is combined with Proposition 2 in Cox, Ingersol, and Ross (1981), hereafter CIR, to develop an equilibrium relationship for futures prices which is then applied to stock index futures. Essentially, we use the asset pricing model to value the cashflow in the arbitrage relation of Proposition 2. Using an arbitrage argument, CIR show that a futures price, at time t for a contract that matures at t+s, is equal to the value of the following cashflow at maturity:

$$P_{t+s}\left[\prod_{j=0}^{s-1}(1+R_{t+j})\right], \tag{1}$$

where P_{t+s} is the price at maturity of the good or asset on which the contract is written and R_t is the interest rate from t to t+1. The one-period interest rate enters because the arbitrage argument uses borrowing and lending at the one-period (one-day) rate to handle the cashflows that arise because of daily settlement.

The next step is to value the cashflow. CIR, in their Section 4, examine futures prices in a continuous-time, continuous-state model. Here, we apply a discrete-time model similar to the models studied by Lucas (1978), Grossman and Shiller (1981), Hansen and Singleton (1982), and Dunn and Singleton (1983). In this model, we assume a representative individual whose actions characterize aggregate economic behavior, and this individual makes consumption and investment decisions by solving the following multiperiod optimization problem:

$$J(w_t) = \max_{\{c_{t+1}\}\{\underline{a}_{t+1}\}} E_t[\sum_{j=0}^{\infty} \beta^j U(c_{t+j+1}, c_{t+j}, c_{t+j-1}, \ldots)]$$

$$= \max_{\{c_{t+1}\}\{\underline{a}_{t+1}\}} E_t[U(c_{t+1}, c_t, c_{t-1}, \ldots) + \beta J(w_{t+1})],$$

subject to the standard budget constraints. c_t is real consumption, $\underline{a}_t$ is a vector of shares held in different assets, and w_t is real wealth. E_t is the conditional expectations operator and the conditioning set is information available at the end of time t. Here, we do not require the utility function to be time-additive.[4] At the end of period t, the individual allocates wealth between consumption for period t+1 and shares that will be held during t+1. Trading takes place at equilibrium prices. The individual takes asset prices as given exogenously, but his(her) optimizing behavior requires asset prices to satisfy the following relationship:

$$p_{it} = E_t[\frac{\beta J'(w_{t+1})}{J'(w_t)} (p_{i,t+1} + x_{i,t+1})], \qquad (2)$$

where $J'(\cdot)$ is the marginal utility of real wealth, p_{it} is the real price of asset i at the end of period t, and x_{it} is the real dividend

or cashflow received for holding asset i during t. Equation (2) can be solved recursively to produce the following relationship:

$$p_{it} = \sum_{j=1}^{\infty} \beta^j E_t\left[\frac{J'(w_{t+j})}{J'(w_t)} x_{i,t+j}\right]. \tag{3}$$

This valuation model simply states that the value of an asset is equal to the expected value of its future cashflows weighted by the corresponding marginal utility of wealth. This relationship can be applied to any asset and can be easily applied to value a single cashflow as in (1). In the case of a bond or an asset with a finite life, the infinite sum becomes a finite sum that stops with the maturity of the asset.

In this model, prices (or values), wealth, dividends and cashflows, and consumption are all denominated in consumption units because individuals are optimizing the utility of real consumption; all variables are real quantities as opposed to nominal or dollar quantities. To convert to a valuation model in nominal terms, we first define the nominal cashflows and prices:

$$x_{it} \equiv D_t X_{it}$$

$$p_{it} \equiv D_t P_{it},$$

where X_{it} and P_{it} are the nominal dividends and prices, respectively, and D_t is the consumption price deflator (or the reciprocal of the consumption price index). We then define a new variable, $\lambda_t \equiv D_t J'(w_t)$, which is the product of the marginal utility of real wealth and the consumption price deflator, and substitute these expressions into equation (3) to get an asset pricing model in nominal terms:[5]

$$P_{it} = \sum_{j=1}^{\infty} \beta^{j} E_t\left[\frac{\lambda_{t+j}}{\lambda_t} X_{i,t+j}\right]. \tag{4}$$

The model in (4) can now be used to value cashflows in nominal terms. In some cases, we do not need estimates of aggregate consumption and consumer prices to apply the model empirically. We use it to value the single cashflow in (1) to get

$$H_t(s) = E_t\left[\frac{\beta^{s} \lambda_{t+s}}{\lambda_t} P_{t+s}\left(\prod_{j=0}^{s-1}(1+R_{t+j})\right)\right], \tag{5}$$

where $H_t(s)$ is the futures price at time t for a contract that matures at (t+s). If P_t represents the value for a portfolio of stocks or a stock index, then $H_t(s)$ is the price for the stock index futures. We also use the asset pricing model in nominal terms to derive an equilibrium relationship for nominal risk-free interest rates including the one-period rates R_t. Let $B_t(t+k)$ be the price of a default-free discount bond that matures at time (t+k) paying \$1, then

$$B_t(t+k) = E_t\left(\frac{\beta^{k} \lambda_{t+k}}{\lambda_t}\right).$$

For one-period nominal interest rates, we have

$$B_t(t+1) = \frac{1}{1+R_t} = E_t\left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right).$$

The equilibrium pricing relationship for stock index futures in (5) is not very useful in its present form. From that equation, one can explore the conditions for the futures price to be above or below the expected value of the future level of the index, but empirically testable implications are difficult to derive. To derive some testable implications, we add the assumption that log stock price changes, $\Delta \ell n P_t$, and

changes in the marginal utility of wealth variable, $\Delta \ln \lambda_t$, are part of a stationary multiple time series representation with normally distributed innovations:[6]

$$\Delta \ln P_t = \overline{P} + \sum_{j=0}^{\infty} \underline{b}_j' \, \underline{\varepsilon}_{t-j}$$

$$\Delta \ln \lambda_t = \overline{\lambda} + \sum_{j=0}^{\infty} \underline{a}_j' \, \underline{\varepsilon}_{t-j}.$$

The innovations have mean zero and a covariance matrix $\Omega = E(\underline{\varepsilon}_t \underline{\varepsilon}_t')$. From equation (5), we evaluate the following moment generating function:

$$\frac{H_t(s)}{P_t} = E_t[\exp\{(\ln P_{t+s} - \ln P_t) + \sum_{j=0}^{s-1} \ln(1+R_{t+j}) + s\ln\beta + (\ln\lambda_{t+s} - \ln\lambda_t)\}] \tag{6}$$

Noting that the one-period interest rates are related to the conditional expectations of changes in the marginal utility of wealth variable, we make the following substitutions.

$$\sum_{j=0}^{s-1} \ln(1+R_{t+j}) = - \sum_{j=0}^{s-1} \ln\{E_{t+j}[\frac{\beta\lambda_{t+j+1}}{\lambda_{t+j}}]\}$$

$$= - \sum_{j=0}^{s-1} \ln\{E_{t+j}[\exp\{\ln\beta + \Delta\ln\lambda_{t+j+1}\}]\}$$

$$= -[s\ln\beta + s\overline{\lambda} + \frac{s}{2}\underline{a}_0'\Omega\underline{a}_0 + \sum_{j=1}^{s} \sum_{k=1}^{\infty} \underline{a}_k' \underline{\varepsilon}_{t+j-k}]$$

$$\ln\lambda_{t+s} - \ln\lambda_t = \sum_{j=1}^{s} \Delta\ln\lambda_{t+j} = s\overline{\lambda} + \sum_{j=1}^{s} \sum_{k=0}^{\infty} \underline{a}_k' \underline{\varepsilon}_{t+j-k}$$

Combining these two expressions, we get

$$s\ell n\beta + (\ell n\lambda_{t+s} - \ell n\lambda_t) + \sum_{j=0}^{s-1} \ell n(1+R_{t+j}) = -\frac{s}{2}\underline{a}_0'\Omega\underline{a}_0 + \sum_{j=1}^{s}\underline{a}_0'\underline{\varepsilon}_{t+j}.$$

This expression is substituted into (6) and we get

$$\ell n(\frac{H_t(s)}{P_t}) = E_t(\ell nP_{t+s} - \ell nP_t) + \frac{1}{2}\mathrm{Var}_t(\ell nP_{t+s} - \ell nP_t)$$

$$+ \mathrm{Cov}_t[\ell nP_{t+s} - \ell nP_t), \sum_{j=1}^{s}\underline{a}_0'\underline{\varepsilon}_{t+j}],$$

where Var_t and Cov_t are the conditional variances and covariances, respectively. Now we examine the change in the log of the futures price:

$$\Delta\ell nH_t(s) \equiv \ell nH_t(s) - \ell nH_{t-1}(s) = E_t(\ell nP_{t+s}) - E_{t-1}(\ell nP_{t+s})$$

$$+ \frac{1}{2}\mathrm{Var}_t(\ell nP_{t+s} - \ell nP_t) - \frac{1}{2}\mathrm{Var}_t(\ell nP_{t+s} - \ell nP_{t-1})$$

$$+ \mathrm{Cov}[(\ell nP_{t+s} - \ell nP_t), \sum_{j=1}^{s}\underline{a}_0'\underline{\varepsilon}_{t+j}]$$

$$- \mathrm{Cov}[(\ell nP_{t+s} - \ell nP_{t-1}), \sum_{j=1}^{s+1}\underline{a}_0\underline{\varepsilon}_{t-1+j}].$$

The expression for $E_t(\ell nP_{t+s}) - E_{t-1}(\ell nP_{t+s})$ is evaluated by applying the rules for revising forecasts for a fixed future period found in Nerlove, Grether, and Carvalho (1979, p. 88). The conditional variances and covariances are separately evaluated and we arrive at the following equation:

$$\Delta\ell nH_t(s) = (\sum_{j=0}^{s}\underline{b}_j)'\underline{\varepsilon}_t - \frac{1}{2}(\sum_{j=0}^{s}\underline{b}_j)'\Omega(\sum_{j=0}^{s}\underline{b}_j) - (\sum_{j=0}^{s}\underline{b}_j)'\Omega\underline{a}_0. \quad (7)$$

The first term in equation (7) is a linear combination of the innovations for the current period, hence this term is a random variable which is independent of the past. The last two terms are not random as

they are functions of the parameters of the multiple time series for stock prices and the marginal utility of wealth variable. The series $\Delta \ell n H_t(s)$ will resemble a serially uncorrelated process if the changes in the last two terms as we approach maturity are small relative to the variation in the first term. This is precisely the case one would anticipate for stock index futures. Because stock prices experience much variation and resemble random walks, it is reasonable to conjecture that the coefficients $\underline{b}_1$, $\underline{b}_2$, ... are quite small in absolute value relative to the coefficients in $\underline{b}_0$.[7] But s in the summations decreases as we approach maturity and over time we can have variations in the last two terms of (7) and changes in the variance of the random term. The last two terms would exhibit little change though and the variation in $\Delta \ell n H_t$ would be dominated by the random variation of the first term; hence, the price changes should be very close to white noise around a nonzero mean. LeRoy (1982) has noted that in models with risk aversion the martingale property does not generally hold for futures prices, but in this model with risk aversion and stochastic interest rates, the log-price changes on stock index futures are very close to being serially uncorrelated. This observation suggests that an analyst may not be able to detect any serial correlation in the price changes empirically, but tests using filter rules and other techniques may be influenced by the variation in the two nonrandom terms.

The last term in equation (7) represents the risk premium in the price changes. If $\underline{a}_0 = \underline{0}$, then the futures price, H_t, is a martingale and satisfies the requirements of simple expectation models. The sign and the size of the risk premium depends, as in other models, on the

conditional covariance of spot prices with the marginal utility of wealth variable. The risk premium can be written as $\sum_{j=0}^{s} \underline{b}_j'\Omega\underline{a}_0$, which is the conditional covariance between $\Delta\ell n P_{t+s}$ and a sequence of one-period forecast errors for the marginal utility of wealth variable. By the argument in the previous paragraph, we assert that for stock index futures, $\underline{b}_0'\Omega\underline{a}_0$ will dominate the rest of the summation in the risk premium. $\underline{b}_0'\Omega\underline{a}_0$ is simply the conditional covariance between the one-period forecast errors for log-stock price changes and the change in the log of the marginal utility of wealth variable; this term is the analog of the local covariance in continuous-time models. With risk aversion, the marginal utility of real wealth is a declining function of real wealth. If there is no consumption price inflation (or deflation), then $\underline{b}_0'\Omega\underline{a}_0$ is the conditional covariance between real stock prices and marginal utility of real wealth. Since a large portfolio of stocks comprises a significant portion of wealth, we would expect a positive covariance between real stock prices and real wealth and a negative covariance between real stock prices and the marginal utility of real wealth. When real stock prices turn out to be greater than expected, real wealth will tend to also be greater than expected. Hence we conjecture that the terms $\underline{b}_0'\Omega\underline{a}_0$ and $(\sum_{j=0}^{s} \underline{b}_j)'\Omega\underline{a}_0$ are negative, which results in the futures price being less than the expected value of the spot price to prevail at maturity (normal backwardation). Alternatively, we can examine the special case of inflation with log utility. From Kraus and Litzenberger (1975), we know that the optimal return function $J(w_t)$ will have the form $a+b\ell n w_t$ if $u(c_t) = \ell n c_t$. In this case, $J'(w_t)$ will be w_t^{-1} and $\Delta\ell n\lambda_t$ will be a negative function of the change in the log of nominal

wealth. $\underline{b}_0'\Omega\underline{a}_0$ will be the negative of the conditional covariance between nominal stock price changes ($\Delta \ell n P_t$) and changes in nominal wealth ($\Delta \ell n W_t$). Along the lines of our previous argument, we would expect a positive covariance between nominal stock price changes and nominal wealth. The resulting risk premium should be positive. These two cases lead us to believe that the risk premium for price changes on stock index futures will have the normal backwardation property.

Samuelson (1965) has argued that the variation of futures prices will change as the contract approaches maturity; in fact, he argued that the volatility should increase as the contract approaches maturity, which at first seems counterintuitive. Rutledge (1976), however, has shown that the variance of futures prices will remain constant if the spot price follows a random walk; Samuelson's result applies when the process on the spot price is stationary. In our model, the stock price is a non-stationary process and the variance of the futures price and the risk premium are constant if stock prices follow a random walk. If stock prices are not a pure random walk (with or without drift), then the variance will change as we approach maturity, but we cannot predict the direction of the change without further information.

From the model in equation (7), we have several hypotheses that can be examined empirically by using actual log price changes on stock index futures: (1) there may be nonzero risk premia which may vary, (2) the volatility or variance of price changes may vary as we approach maturity, and (3) as a result a time series, $\Delta \ell n H_t$, constructed from prices on near contracts may have some periodicity due to the dependence of the risk premium and the variance on time to maturity. In the next section,

we present empirical evidence on these hypotheses by studying the behavior of prices on the Standard & Poor's 500 futures, the New York Stock Exchange Composite futures, and the Kansas City Value Line futures, hereafter, the S&P 500, the NYSC, and the KCVL, respectively. These three index futures are studied because they are the most popular and most actively traded of the Stock index futures. Before presenting the statistical analysis, we offer some casual empirical evidence on the risk premium in stock index futures.

If the martingale model is a good approximation for stock index futures, then we have a good measure for the market's expected level of the index to prevail at maturity of the contract. Given an estimate of the expected dividends on the stocks in the index, we can then compute the expected return for the index. Expected returns on major stock market indices are frequently used as proxies for the expected return on the market portfolio. Do futures prices for the major stock indices produce realistic numbers for the expected returns on common stock portfolios? Standard & Poor's publishes a quarterly dividend series for its index of 500 common stocks, adjusted to the level of the index. The numbers are reported as four-quarter moving totals, but one can use the numbers in the technical appendix to LeRoy and Porter (1981) to recover the quarterly dividend series. The dividend series for 1980:I to 1984:I is reproduced in Table I. Note the stability of the series. Clearly, dividends over a one-quarter time horizon are very predictable. Although, the market may not be able to predict perfectly dividends over the next quarter, the forecasts of market participants must be quite accurate, and the range for expected dividends must certainly fall in

a narrow band around the actual levels. It is highly unlikely that expected quarterly dividends for the S&P 500 were ever as high as 2.0 during this period. We have divided the period June 17, 1982, to March 15, 1984, into seven separate thirteen-week periods, each terminating with the expiration of a futures contract on the S&P 500. We use the futures price at the beginning of each period as our measure of the expected level of the index for the end of the period. We then calculate two estimates for the expected return: one using the actual dividends during the quarter as a measure of expected dividends, and a second estimate using a high estimate of 2.0 for expected dividends in every period. The numbers are then compared to the risk-free return measured by the rate of return on holding a thirteen-week Treasury bill over the same period.[8] The numbers are summarized in Table II.

During the first four periods, estimates for expected returns are less than the corresponding risk-free returns. There were times when the futures prices were less than the current spot prices. During the last three periods, the estimated expected returns are higher than the risk-free rates, but nowhere near the levels that we would consider appropriate for expected returns on risky assets. Most estimates of the market risk premium indicate that the expected return on a large portfolio of stocks, on an annual basis, is 8 to 9% higher than the return on risk-free securities.[9] On a quarterly basis, this difference would be around 2%. Expected dividends would need to be more than twice their actual level to get a quarterly market risk premium of 2% for the last period shown in Table II. This casual empirical evidence implies that there is a risk premium and that futures prices must be significantly less than the corresponding expected spot prices for the S&P 500.

III. Empirical Analysis

In this section we examine some of the empirical implications of the model for stock index futures. Specifically, we explore whether there are risk premia and whether the risk premia and the variances change as we approach maturity. The standard model for analyzing futures prices is one in which price changes are independent of past price changes and the variance is constant. In many studies, the possibility of a changing variance is ignored, but this issue has been examined by Rutledge and others for commodity futures.[10] First, we simplify the model for $\Delta \ell n H_t(s)$ because we examine data on stock index futures only and we do not attempt the difficult task of formulating a multiple time series model.

$$\Delta \ell n H_t(s) = \sigma(s)\ e_t - \frac{1}{2}\sigma^2_{(s)} + \mu(s),$$

where e_t is a standard normal random variable and is serially independent. $\mu(s)$ and $\sigma^2_{(s)}$ are the mean and variance parameters which depend on time to maturity. $\mu(s)$ also represents the risk premium. We then examine four hypotheses: (1) changing risk premia and changing variances, (2) no risk premium and changing variances, (3) a constant risk premium and a constant variance, and (4) no risk premium and a constant variance. One implication of the model is that the risk premium must be constant if the variance is constant.

The model is applied to weekly changes in the log of prices for the S&P 500, the NYSC, and the KCVL futures. Thursday settlement prices are used to measure prices.[11] Weekly price changes, instead of daily

price changes, are studied so that we can avoid the weekend and day-of-the-week effects which have been found in stock returns. Because most of the activity (open interest and trading volume) has been in the near contracts, we focus the analysis on the near contracts only. For each of the three futures, we have constructed a time series of log price changes on the near contract. We start with the log price change on the nearest contract and follow it to one week before maturity, then for the following week we pick up the log price change on the next contract which always has thirteen weeks to maturity during the sample period. The series run from the beginning of trading in 1982 up to February, 1984. The series for the S&P 500 and the NYSC have 91 observations and the series for the KCVL has 104 observations. The series contain 13 different times to maturity so that hypothesis (1) has 13 risk premia and 13 variances, or 26 parameters to estimate. The three remaining hypotheses impose restrictions on the risk premia and variances and are therefore testable.

The parameters are estimated by the method of maximum likelihood and the likelihood ratio statistic, $-2\ell n\lambda$, is used to test the various restrictions. Using the assumption that the innovations are normally distributed, we can write the log-likelihood function for the most general model (hypothesis 1) as follows:

$$\ell nL = -\sum_{s=1}^{13} \left\{\frac{T_s}{2}\,\ell n\sigma^2_{(s)} + \frac{1}{2\sigma^2_{(s)}} \sum_{t-1}^{T_s} \left(y_{st} - \mu(s) + \frac{1}{2}\,\sigma^2_{(s)}\right)^2\right\},$$

where we have omitted the proportionality constant $-\frac{T}{2}\,\ell n(2\pi)$. T_s represents the number of observations for a given time to maturity, y_{st} represents the observations on $\Delta\ell nH_t(s)$, and T is the summation of

T_s, s=1,...,13. The estimates under hypothesis 1 are presented in Table III. Only a few of the mean parameters are statistically significant, and we cannot detect any particular pattern in the risk premia or the variance estimates. If we exclude the mean parameters for one week to maturity on the NYSC and the KCVL contracts, it appears that the risk premia are smaller in absolute value for the last six weeks before maturity. The more interesting results involve the tests of the restrictions, and these results are shown in Tables IV, V, and VI. For each series, we conduct five tests using the likelihood ratio statistic. For all three futures, we reject the hypothesis of no risk premium and a constant variance at standard significance levels. For every test of a restrictive hypothesis (2, 3, or 4) against hypothesis 1, we reject the restrictive hypothesis at standard significance levels. It is interesting to note that if we test for a risk premium in a model with a constant variance, we do not reject the null hypothesis of no risk premium, and we accept the null hypothesis of a constant variance in a model with no risk premium. These restrictions, however, are rejected when they are tested against the model of Section II. We have also computed several test statistics to check for serial correlation in the time series, but a detailed report is omitted here. None of the tests indicate any evidence of serial correlation, either before or after a correction for the periodic components.

The likelihood ratio tests that we use are based on the asymptotic chi-squared distributions for likelihood ratio statistics. As we noted before, the total sample sizes are 91 for the S&P 500 and the NYSC and 104 for the KCVL. One might naturally inquire about the accuracy of

large-sample tests in these applications. In many cases, it is impossible to assess the accuracy of a large-sample approximation for test statistics, but we can develop a small-sample test of this model and compare the results to those for the likelihood ratio tests.

The model suggests that log price changes on stock index futures will be periodic: the mean and variance parameters will depend on time to maturity. One method of testing for periodicity is to test for a constant mean across the thirteen different times to maturity by applying the F statistic in analysis of variance. Under the null hypothesis of a constant mean and constant variance, the F statistic has an F distribution in finite samples.[12] We can also compute the likelihood ratio statistic for a test of constant means and compare the results to those for the F statistic.

This test is essentially a test of one risk premium and one variance versus thirteen risk premia and a constant variance. The hypothesis of thirteen risk premia and a constant variance is not consistent with the model, but it does allow us to construct a finite sample test. If the F test rejects the null hypothesis, we are not certain whether rejection is caused by different risk premia or different variances or both. The model suggests that rejection would be caused by both. The likelihood ratio statistics and the F statistics are reported in Table VII. For the S&P 500, the F statistic has a marginal significance level of 7.74% and the likelihood ratio statistic, based on the chi-squared approximation, has a marginal significance level of 4.47%. For the NYSC, the F statistic has a marginal significance level of 8.23% and the likelihood ratio statistic has a marginal significance level of 4.8%. For the KCVL,

the F statistic and the likelihood ratio statistic have marginal significance levels of 12.51% and 9.24%, respectively. Although the likelihood ratio test is not exact, the large-sample approximations do produce results which are very similar to those for the F test. The F statistics, alone, indicate some evidence of periodicity in the series for the S&P 500 and the NYSC. The likelihood ratio tests, however, provide more information on those restrictions which are being rejected by the data. Finally, the more interesting tests yield much stronger results than do the likelihood ratio tests which are used for comparisons with the F tests.

IV. Summary and Conclusions

We have examined the behavior of price changes on stock index futures within a model with risk aversion and stochastic interest rates, and we find that the log price changes should closely resemble a white noise process, but with subtle deviations resulting from time-varying risk premia and variances. We then test these results by using weekly price changes on stock index futures. The empirical results indicate strong support for the results of the model and reject the restrictions implied by the martingale model and by models with constant variances. In addition, we present some casual empirical evidence which indicates that there are substantial risk premia in the prices for the S&P 500 futures.

The market for stock index futures is one in which we might find hedgers on both sides of the market. A portfolio manager can hedge a large diversified portfolio against a drop in the market by selling an appropriate number of stock index futures, and an investor who anticipates future cashflows that are to be directed into the stock market

can hedge against a price rise by buying stock index futures. An analysis of the potential hedgers will not provide much insight on the risk premium that might develop in this market; besides, the hedgers may even shift between long and short positions. The arbitrage argument in CIR provides a framework for analyzing price formation in stock index futures, and we find that the data support the results which we derive in Section II. Thus, we conclude that prices on stock index futures do contain risk premium and that the variance of price changes varies as we approach maturity.

TABLE I

Quarterly Dividends, S&P 500

Quarter	Dividends
1980:I	1.46
II	1.56
III	1.56
IV	1.58
1981:I	1.58
II	1.67
III	1.69
IV	1.69
1982:I	1.67
II	1.76
III	1.73
IV	1.71
1983:I	1.71
II	1.79
III	1.79
IV	1.80
1984:I	1.80

SOURCE: Standard & Poor's, Statistical Service, Current Statistics, May 1984.

TABLE II

Period	Risk-Free Return	Expected Return, S&P 500: Perfect Foresight on Dividends	A High Estimate for Expected Dividends
6/17/82 - 9/16/82	3.271%	-0.669%	-0.418%
9/16/82 - 12/16/82	2.064	1.204	1.438
12/16/82 - 3/17/83	2.011	1.752	1.966
3/17/83 - 6/16/83	2.101	1.036	1.177
6/16/83 - 9/15/83	2.230	2.247	2.371
9/15/83 - 12/15/83	2.349	2.415	2.537
12/15/83 - 3/15/84	2.370	2.895	3.018

TABLE III

Maximum Likelihood Estimates

S&P 500:

s	$\mu(s)$	$\sigma^2(s)$
13	-.006258	.0008064
12	.023457	.0002410
11	-.010858	.0001741
10	.022695	.0009187
9	.014677	.0003298
8	.007747	.0003609
7	-.020746	.0005536
6	-.001984	.0010953
5	.002364	.0003096
4	-.001189	.0009649
3	.008380	.0011996
2	.008054	.0002427
1	-.003937	.0001586

Log-Likelihood = 305.1439 T = 91

NYSC:

s	$\mu(s)$	$\sigma^2(s)$
13	-.011516	.0001869
12	.023391	.0009110
11	.014384	.0003053
10	.007521	.0003752
9	-.020417	.0005161
8	-.004075	.0011544
7	.004517	.0003013
6	-.000296	.0010244
5	.008472	.0012130
4	.008653	.0002379
3	-.004365	.0001781
2	-.006249	.0005892
1	.019525	.0001191

Log-Likelihood = 308.1984 T = 91

KCVL:

s	$\mu(s)$	$\sigma^2(s)$
13	-.007850	.0002826
12	.025568	.0007445
11	.013197	.0003560
10	.004166	.0003921
9	-.018229	.0004020
8	.002721	.0011432
7	-.000486	.0010182
6	.005081	.0007763
5	.003991	.0012048
4	.006096	.0003570
3	-.005319	.0002153
2	-.004768	.0006372
1	.018414	.0002129

Log-Likelihood = 342.8729 T = 104

TABLE IV

Hypothesis Tests, S&P 500

		Parameters to Estimate	Log-Likelihood
H_1:	Different risk premia, different variances	26	305.1439
H_2:	No risk premium, different variances	13	291.6364
H_3:	One risk premium, one variance	2	284.0105
H_4:	No risk premium, one variance	1	283.3062

Test of H_2 vs. H_1

$$-2\ell n\lambda = \chi^2(13) = 27.02^{**}$$

Test of H_3 vs. H_1

$$-2\ell n\lambda = \chi^2(24) = 42.27^{**}$$

Test of H_4 vs. H_1

$$-2\ell n\lambda = \chi^2(25) = 43.68^{**}$$

Test of H_4 vs. H_2

$$-2\ell n\lambda = \chi^2(12) = 16.66$$

Test of H_4 vs. H_3

$$-2\ell n\lambda = \chi^2(1) = 1.41$$

NOTES: *Significant at 1% level
**Significant at 2.5% level
***Significant at 5% level

TABLE V

Hypothesis Tests, NYSC

		Parameters to Estimate	Log-Likelihood
H_1:	Different risk premia, different variances	26	308.1984
H_2:	No risk premium, different variances	13	293.1938
H_3:	One risk premium, one variance	2	285.6641
H_4:	No risk premium, one variance	1	285.0278

Test of H_2 vs. H_1

$$-2\ell n\lambda = \chi^2(13) = 30.01^*$$

Test of H_3 vs. H_1

$$-2\ell n\lambda = \chi^2(24) = 45.07^*$$

Test of H_4 vs. H_1

$$-2\ell n\lambda = \chi^2(25) = 46.34^*$$

Test of H_4 vs. H_2

$$-2\ell n\lambda = \chi^2(12) = 16.33$$

Test of H_4 vs. H_3

$$-2\ell n\lambda = \chi^2(1) = 1.27$$

TABLE VI

Hypothesis Tests, KCVL

	Parameters to Estimate	Log-Likelihood
H_1: Different risk premia, different variances	26	342.8729
H_2: No risk premium, different variances	13	330.2635
H_3: One risk premium, one variance	2	324.5357
H_4: No risk premium, one variance	1	323.7319

Test of H_2 vs. H_1

$$-2\ell n\lambda = \chi^2(13) = 25.22^{**}$$

Test of H_3 vs. H_1

$$-2\ell n\lambda = \chi^2(24) = 36.67^{***}$$

Test of H_4 vs. H_1

$$-2\ell n\lambda = \chi^2(25) = 38.28^{***}$$

Test of H_4 vs. H_2

$$-2\ell n\lambda = \chi^2(12) = 13.06$$

Test of H_4 vs. H_3

$$-2\ell n\lambda = \chi^2(1) = 1.61$$

TABLE VII

Tests for Periodicity

	S&P 500	NYSC	KCVL
Maximum Likelihood Estimation			
Log-Likelihood			
- 13 risk premia, 1 variance	294.7168	296.2482	334.1536
- 1 risk premium, 1 variance	284.0105	285.6641	324.5357
Likelihood Ratio Statistic, $-2\ell n\lambda=\chi^2(12)$	21.41	21.17	18.84
Approximate Marginal Significance Level	4.47%	4.80%	9.24%
F Statistic	1.72	1.70	1.54
Degrees of Freedom	12,78	12,78	12,91
Marginal Significance Level	7.74%	8.23%	12.51%

FOOTNOTES

[1]For an example of the issues which interest traders and market observers, see "Pricing of Index Futures," by H. J. Maidenberg, New York Times, p. D5.

[2]For a recent discussion of these issues and others, see LeRoy (1982) and Kamara (1982).

[3]For an introduction to this literature, see LeRoy, Merton (1973), Breeden (1979), and Cox, Ingersol, and Ross (1981).

[4]Lucas proves the existence of the optimal return function $J(w_t)$ and the equilibrium price functions for an exchange economy with time-additive utility. We do not attempt such an ambitious task for this model, but we simply assume that the functions exist (probably not in a closed-form) and are well-behaved.

[5]For the case where the utility function is time-additive and $U'(c_t) = \ell n c_t$, the λ_t variable is the reciprocal of per capita consumption in nominal terms.

[6]This specification for stock price changes includes as a special case the random walk model which is frequently used to study stock price changes. By specifying a multiple time series representation, we can derive our results in a relatively general setting. This representation also includes a factor model for the change in the log of stock prices similar to the factor model for returns in the Arbitrage Pricing Theory of Ross (1976, 1977). The normality assumption is required so that we can evaluate the expectation in (5), which becomes a moment generating function.

[7]The random walk model would have $\underline{b_1} = \underline{b_2} = \ldots = \underline{0}$.

[8]The futures prices for all three indices are taken from two sources: the Wall Street Journal and a data tape from MJK Associates, Computerized Commodity Data Sources. The index levels and the bid and ask rates for Treasury Bills are taken from the Wall Street Journal. The returns on 13-week T-bills are computed from the average of the bid and ask; the discount basis rates have been converted to holding period returns.

[9]See the studies by Ibbotson and Sinquefield (1977) and Merton (1980).

[10]See the survey by Kamara, particularly p. 280.

[11]The S&P 500 matures on the third Thursday of the contract month. The last trading day on the NYSC is the next to last trading day of the contract month, and the last trading day on the KCVL is the last trading day of the contract month. With only one exception during the sample period, the last trading days for the NYSC and KCVL all fell on Wednesdays, Thursdays, or Fridays.

[12]We separately constructed a test statistic using the periodogram which allows the variances to change. The resulting F statistic, however, is identical to the standard F statistic in analysis of variance.

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